2025.M31

2025L020G1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

# Leaving Certificate Examination 2025

# Applied Mathematics

## Ordinary Level

## Tuesday 24 June  Afternoon 2:00 - 4:30

400 marks

**Examination Number**

**Date of Birth**

For example, 3rd February 2005 is entered as 03 02 05

**Centre Stamp**

# Instructions

There are ten questions on this paper. Each question carries 50 marks.

Answer any **eight** questions.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. All of your work should be presented in the answer areas, or on the given graphs, networks or other diagrams. Anything that you write outside of these areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in their simplest form, where relevant.

Diagrams are generally not drawn to scale.

Unless otherwise indicated, take the value of $g$, the acceleration due to gravity, to be $9.8$ m s$^{-2}$.

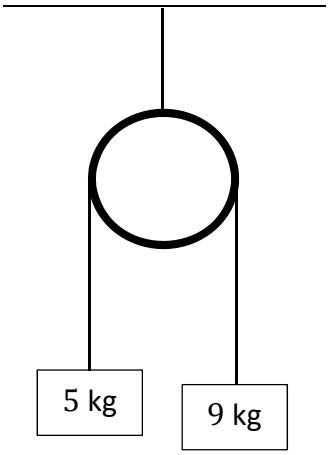
Unless otherwise indicated, $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Write the make and model of your calculator(s) here:

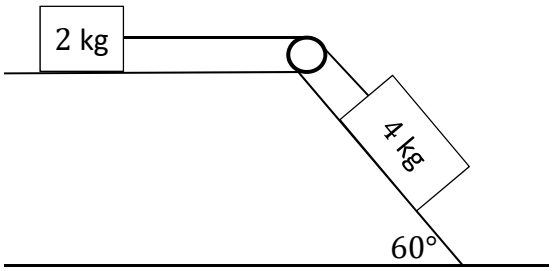
**Question 1**

**(a)** Two blocks of mass $5$ kg and $9$ kg are connected by a light inextensible string which passes over a smooth fixed pulley.

Calculate the tension in the string when the system is released from rest.



**(b)** Two blocks of mass $2$ kg and $4$ kg are connected by a light in extensible string which passes over a smooth fixed pulley.




The $2$ kg block is on a rough horizontal surface. The coefficient of friction between the block and the surface is $\frac{1}{2}$.

The $4$ kg block is on a smooth plane inclined at $60°$, as shown.

The system is released from rest.

**(i)** Draw labelled diagrams to show the forces acting on the blocks when they are moving.

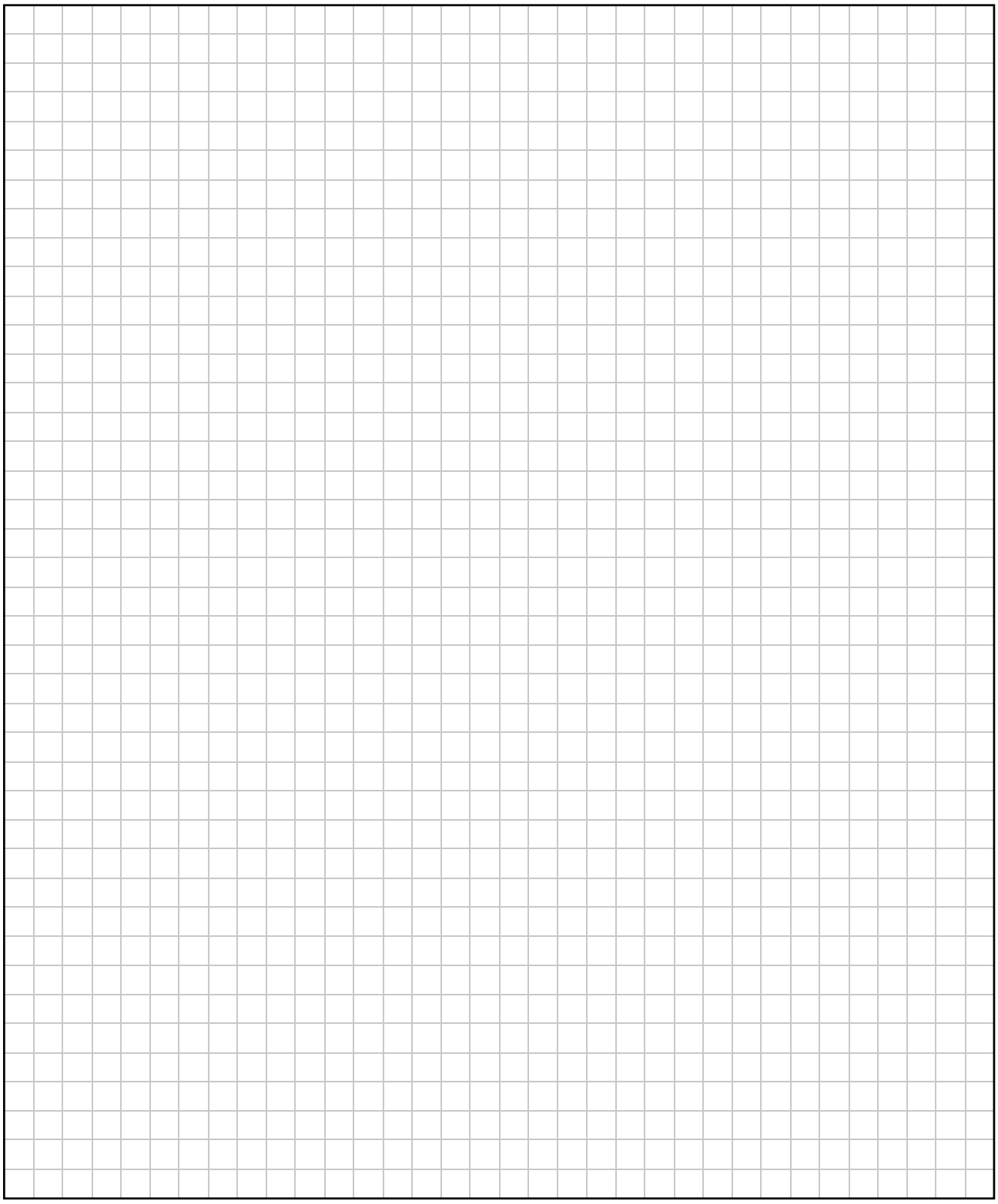
**(ii)** Calculate the common acceleration of the blocks.

**(iii)** Calculate the tension in the string.

**Question 2**

**(a)** A bicycle and its rider have a combined mass of $120$ kg. The bicycle travels across smooth horizontal ground. Air resistance to the motion is estimated to be $50$ N.

**(i)** Calculate the force which the rider needs to apply so as to give the bicycle and herself an acceleration of $2$ m $s^{-2}$.



The rider then cycles up a smooth hill which is inclined at $\alpha$ to the horizontal. Air resistance remains at $50$ N and the rider applies a forward force of $300$ N. The bicycle travels up the hill at constant speed.

**(ii)** Calculate $\alpha$.

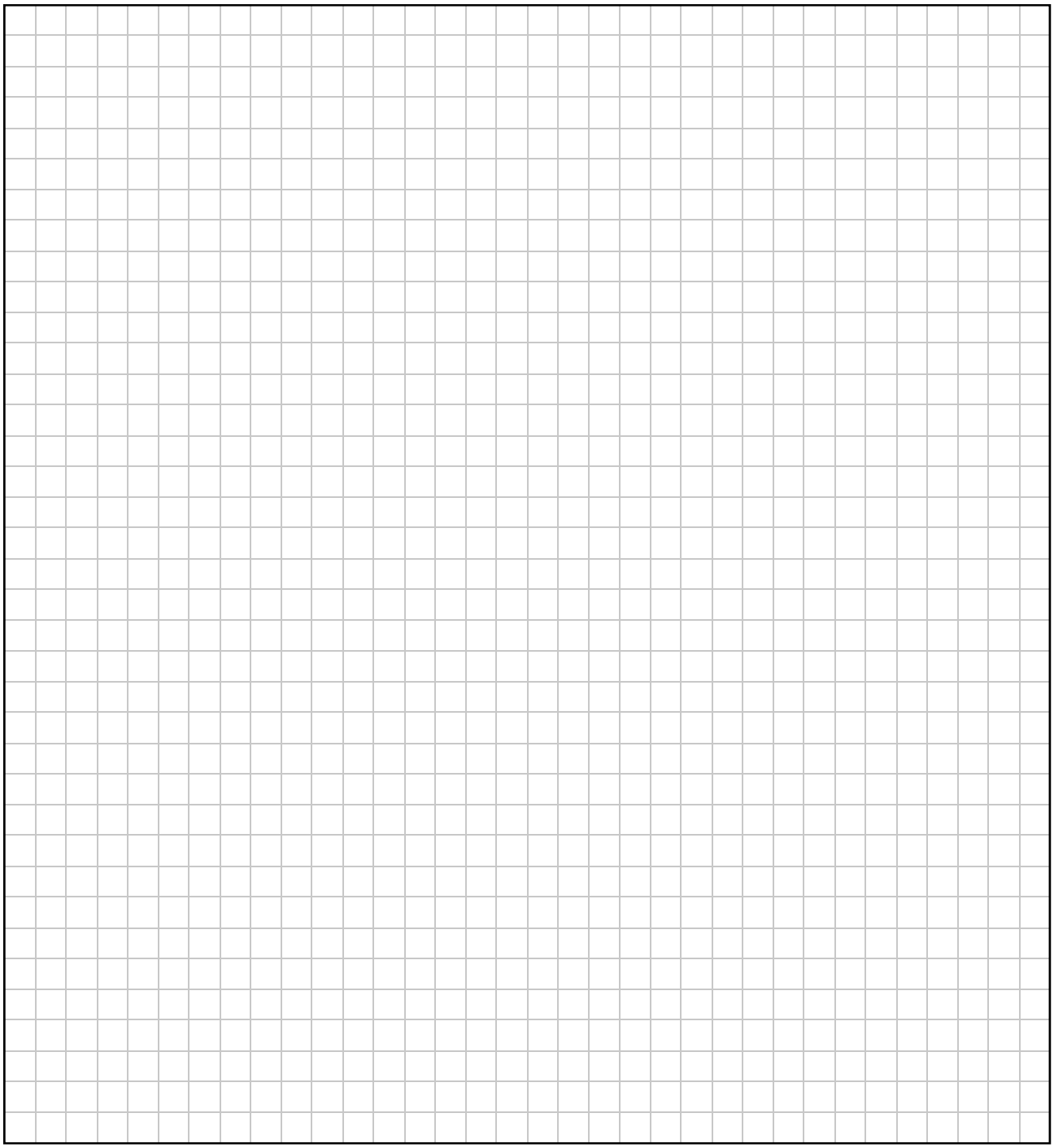
**(b)** A tennis ball is dropped from a height of $3$ m and bounces vertically on a hard surface a number of times. $H$, the maximum height it reaches during each bounce is found to be $75\%$ of the height it previously fell from.

The heights of the ball during each bounce form a geometric sequence which can be represented by the difference equation:

$$H_{n+1} = 0.75H_n$$

where $n \geq 0, n \in \mathbb{Z}$ and $H_0 = 3$ m.

**(i)** Solve the difference equation to find an expression for $H_n$ in terms of $n$.



**(ii)** Calculate $H_4$.

**(iii)** Calculate the total distance travelled by the tennis ball until just before the third bounce.

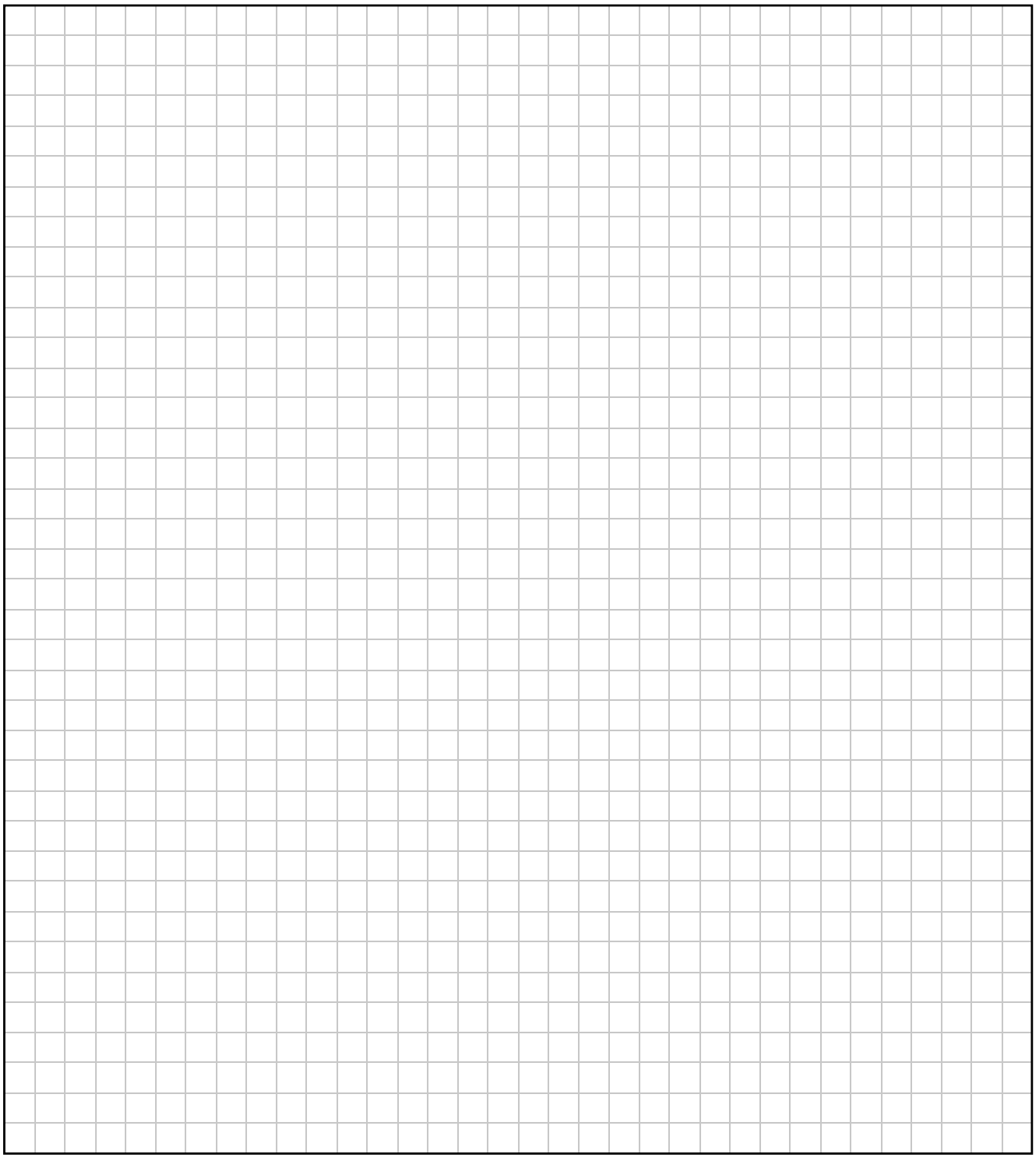
**Question 3**

A car travels along a straight road.

It passes point $A$ at a speed of $15$ m s$^{-1}$ and accelerates uniformly for a time of $5$ s to a speed of $25$ m s$^{-1}$. It maintains this speed of $25$ m s$^{-1}$ for $30$ s. Finally the car decelerates uniformly to rest at point $B$.

$|AB| = 975$ m, the total distance travelled.

**(i)** Calculate the initial acceleration of the car.



**(ii)** Show that the distance the car travels while decelerating is $125$ m.

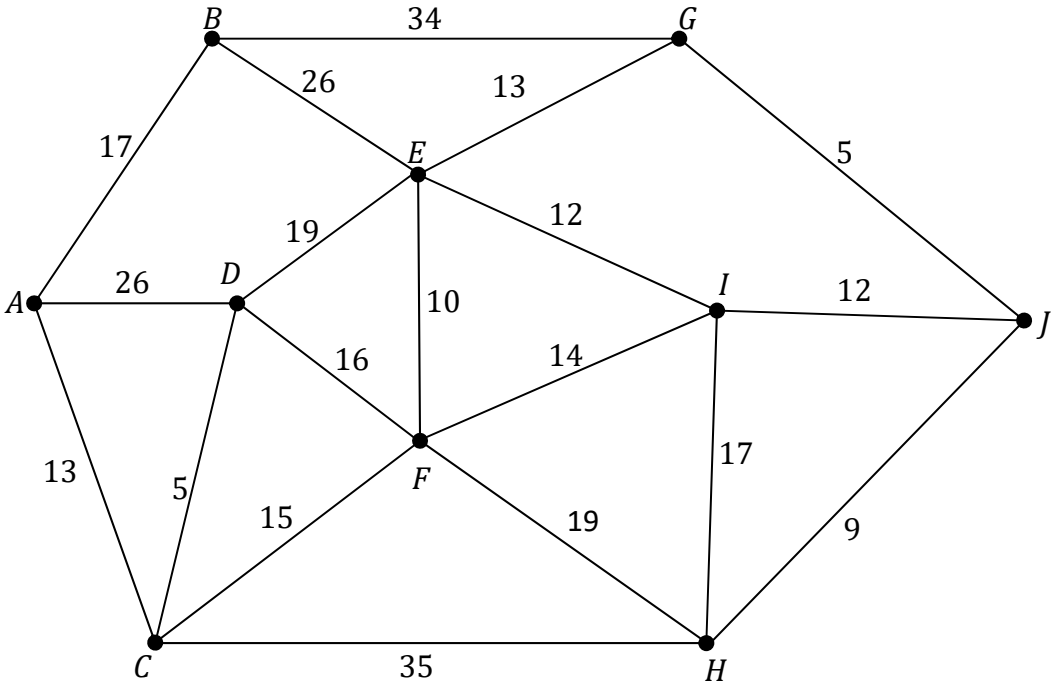
**(iii)** Calculate the deceleration of the car.

**(iv)** Calculate the total time it takes for the car to travel from $A$ to $B$.

**(v)** Calculate the average speed of the car in km hour$^{-1}$ as it travels from $A$ to $B$.

**Question 4**

The diagram below shows a network of hiking trails. The weight of each edge represents the length in km of that part of the trail.




**(i)** Explain what is meant by a path in the context of networks.

**(ii)** Explain what is meant by a cycle in the context of networks.

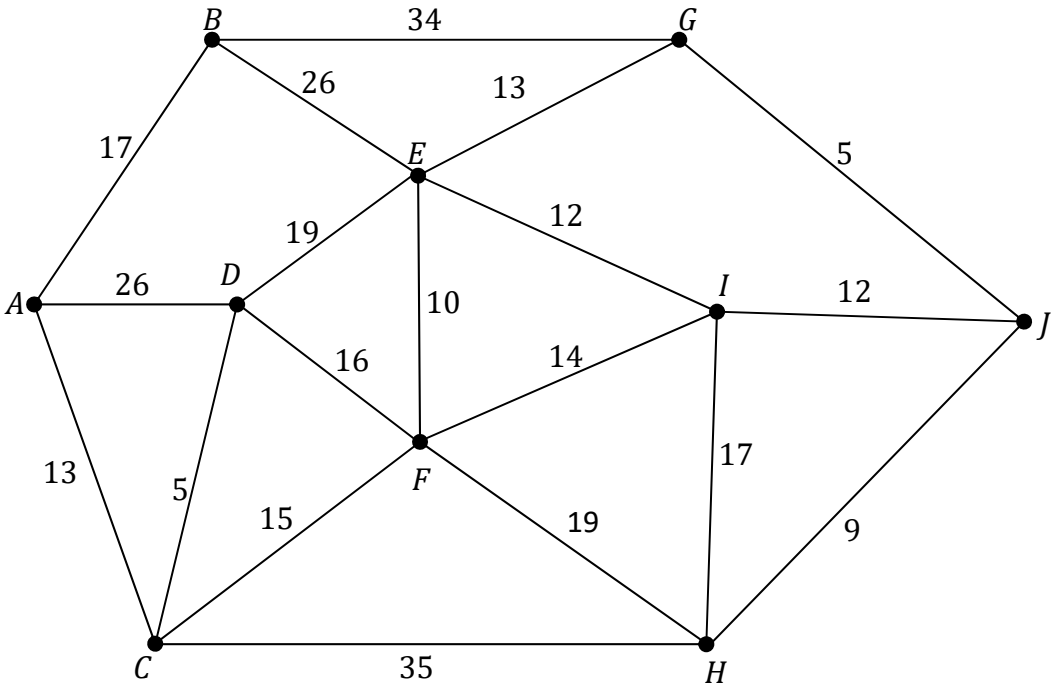
Pádraig wishes to travel from $A$ to $J$ using the shortest path.

**(iii)** Describe Dijkstra's algorithm for finding the shortest path between two nodes in a network.

**(iv)** Use Dijkstra's algorithm to find the shortest path from $A$ to $J$. Write down the shortest path. Calculate the length of the shortest path.

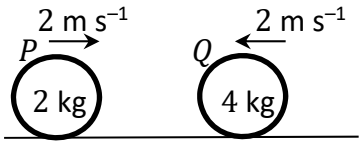
**(v)** Explain what is meant by the minimum spanning tree for a network.

**(vi)** Using Prim's algorithm, find the minimum spanning tree for this network.
Relevant supporting work must be shown.



**Question 5**

**(a)** A smooth sphere $P$ of mass $2$ kg collides directly with another smooth sphere $Q$ of mass $4$ kg on a smooth horizontal table.




$P$ and $Q$ are moving in opposite directions, each with a speed of $2$ m s$^{-1}$.

The impulse imparted to $Q$ due to the collision is $9.6$ N s.

**(i)** Show that the speed of $Q$ after the collision is $0.4$ m s$^{-1}$.

**(ii)** Calculate the speed of $P$ after the collision.

**(iii)** Calculate $e$, the coefficient of restitution for the collision.

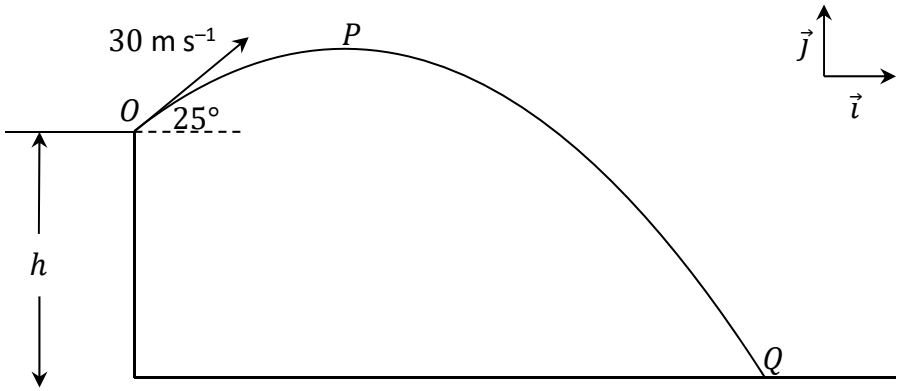
**(b)** A small block is released from rest and slides down a slide of length $4.5$ m which is inclined at $35°$ to the horizontal.


**(i)** Use a suitable model to calculate the speed of the block when it is at the end of the slide.

**(ii)** Outline one assumption made by your model.

**Question 6**

A golf ball is projected from a point $O$ on a cliff with an initial velocity of $30$ m s$^{-1}$ at an angle of $25°$ to the horizontal.




**(i)** Calculate the initial velocity of the ball in terms of $\vec{i}$ and $\vec{j}$.

**(ii)** Calculate the time taken to reach the maximum height at point $P$.

**(iii)** Calculate the displacement of $P$ from $O$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.

$5$ s after it is projected, the ball lands at point $Q$.

**(iv)** Calculate $h$, the height of the cliff.

**(v)** Calculate the distance $|OQ|$.

**Question 7**

**(a)** A ball moves in a horizontal circle of radius $1.5$ m with uniform speed $4.5$ m s$^{-1}$. The ball has a mass of $50$ g.

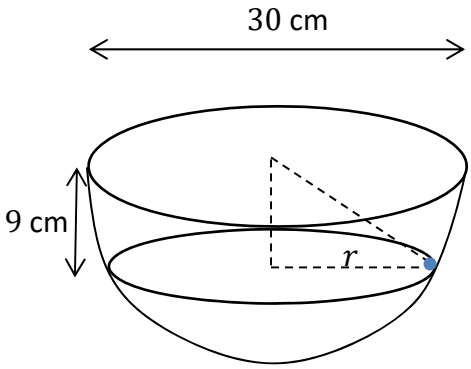
**(i)** Calculate $\omega$, the angular velocity of the ball.

**(ii)** Calculate the centripetal force on the ball.

**(iii)** Calculate the time taken for $6$ revolutions.

**(b)** A small smooth marble of mass $20$ g moves through a horizontal circle of radius $r$ on the inside of a smooth hemispherical bowl of diameter $30$ cm.

The horizontal circle is $9$ cm below the top of the bowl.




**(i)** Calculate $r$.

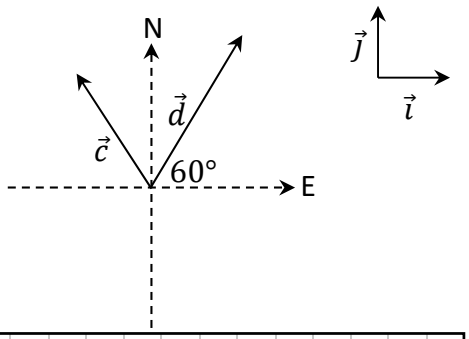
**(ii)** Draw a diagram to show the forces acting on the marble.

**(iii)** Calculate the force which the bowl exerts on the marble.

**Question 8**

**(a)** Displacement vector $\vec{c} = -2\vec{\imath} + 4\vec{\jmath}$ km.

A second displacement vector $\vec{d}$ has a magnitude of $6$ km and a direction $60°$ north of east, as shown in the diagram.




**(i)** Calculate the magnitude and direction of $\vec{c}$.

**(ii)** Express $\vec{d}$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.

**(iii)** Calculate $\vec{c} \cdot \vec{d}$, the dot product of $\vec{c}$ and $\vec{d}$.

**(b)** Seán borrows €$24\,000$ to buy a car. He will repay the loan by paying back equal instalments at the end of each month for $6$ years. The value of each repayment is $A$. Seán is charged an interest rate of $0.7\%$ per month on the loan.

**(i)** Calculate the number of monthly repayments Seán will make over $6$ years.

**(ii)** Show that the value, $Y$, in €, which Seán owes at the end of month $n$ may be modelled by the difference equation:

$$Y_{n+1} = 1.007Y_n - A$$

where $n \geq 0$, $n \in \mathbb{Z}$ and $Y_0 = 24\,000$.

**(iii)** Solve the difference equation to find an expression for $Y_n$ in terms of $n$ and $A$.

**(iv)** Calculate the value of each repayment $A$.

**Question 9**

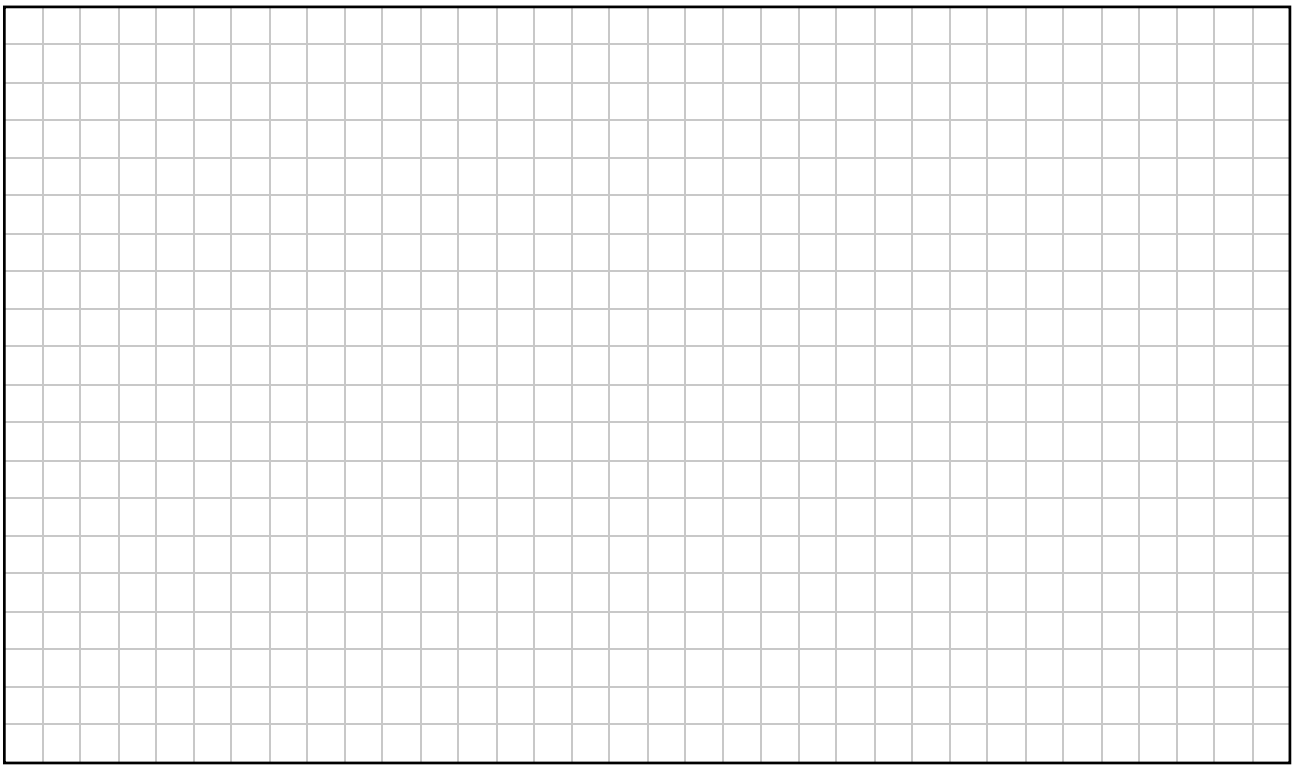
**(a)** The 7 houses in a housing development are labelled with the letters $A, B, C, D, E, F$ and $G$. Some of the houses are connected to each other by pipes.

The table below shows the lengths of the pipes between houses that are connected. The lengths are measured in m.

| | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|
| $A$ | – | 14 | 15 | 16 | 26 | – | – |
| $B$ | 14 | – | – | 4 | 15 | 13 | – |
| $C$ | 15 | – | – | 5 | – | 17 | 27 |
| $D$ | 16 | 4 | 5 | – | 17 | 16 | 25 |
| $E$ | 26 | 15 | – | 17 | – | – | 5 |
| $F$ | – | 13 | 17 | 16 | – | – | 12 |
| $G$ | – | – | 27 | 25 | 5 | 12 | – |

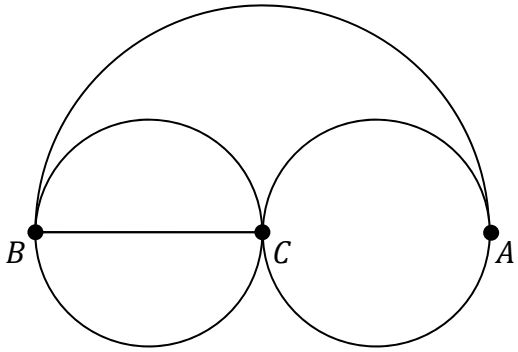
In order for the pipes to be upgraded in the most efficient manner, an engineer wishes to know the shortest length of piping needed so that every house has access to a new shared water supply.

**(i)** Using Kruskal's algorithm, find the minimum spanning tree for the network of houses. Relevant supporting work must be shown.



**(ii)** Calculate the length of piping on your minimum spanning tree.

**(b)** The graph below shows three nodes connected by a number of edges.




**(i)** Write $M$, the adjacency matrix for this graph.

**(ii)** Calculate $M^2$.

**(iii)** What information is provided by the elements of $M^2$?

**Question 10**

**(a)** A business has taken on selling a new product on the internet. In the first month the business sold $8$ of these items. In the second month it sold $10$ of them.

An employee predicts that $P$, the number of items sold in any month, can be predicted using the following second-order difference equation:

$$P_{n+2} = 2P_{n+1} - 0.75P_n$$

where $n \geq 0, n \in \mathbb{Z}, P_0 = 8$ and $P_1 = 10$.

**(i)** Calculate $P_2$.

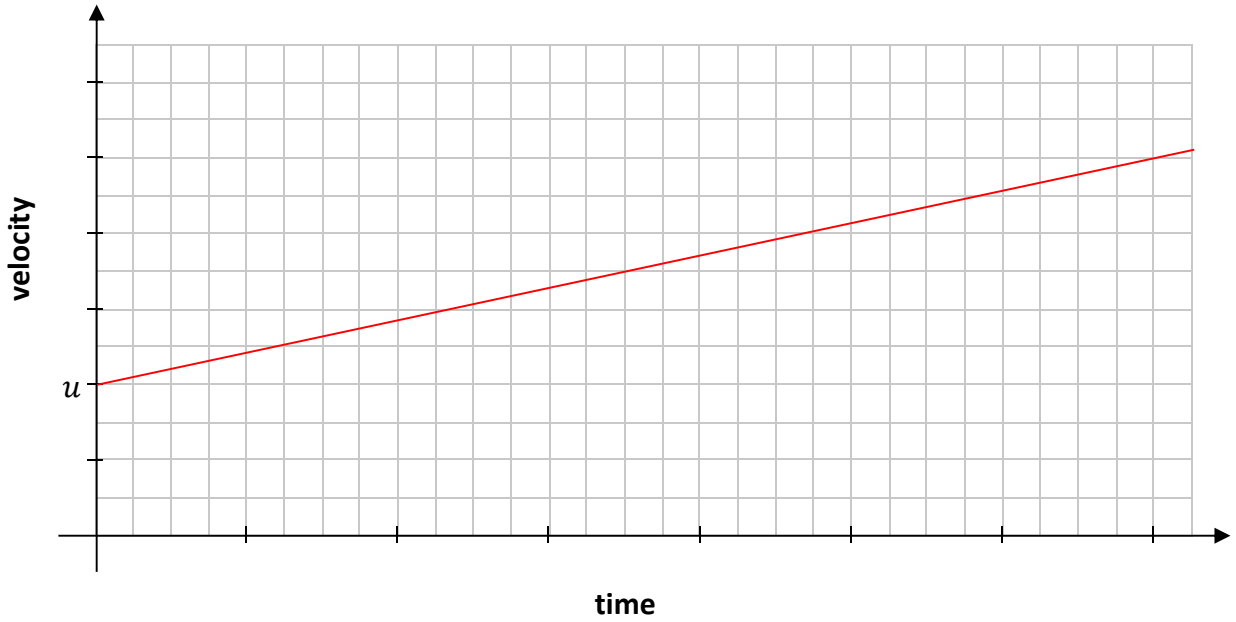
The difference equation has the characteristic quadratic equation $x^2 - 2x + 0.75 = 0$, which can also be written as $4x^2 - 8x + 3 = 0$.

**(ii)** Solve the characteristic equation, i.e. calculate the two roots of the equation.

**(iii)** Hence solve the difference equation to find an expression for $P_n$ in terms of $n$.

**(iv)** Calculate $P_6$ to the nearest whole number.

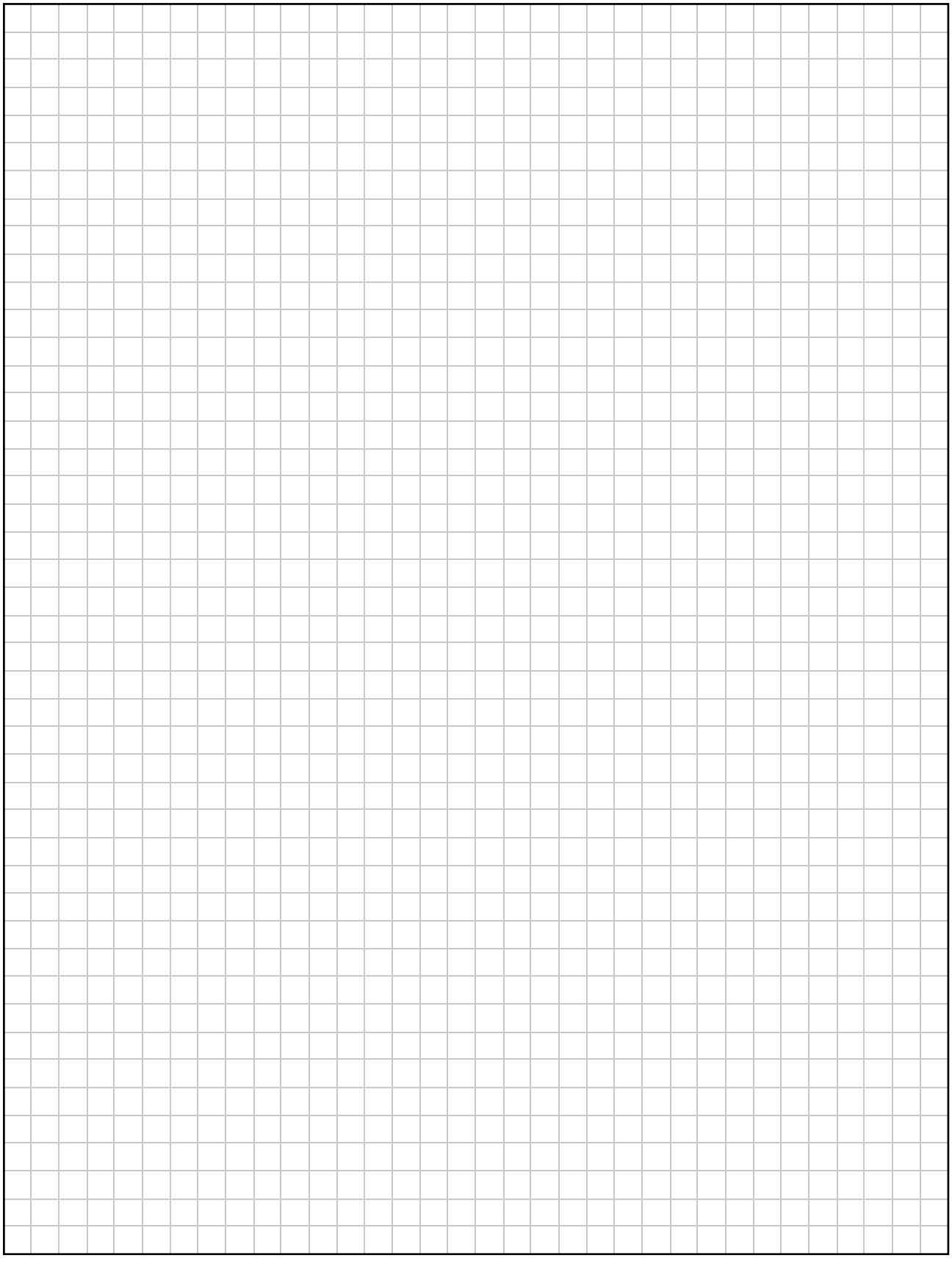
**(b)** The velocity-time graph below is for the motion of a particle which has velocity $u$ when time $t = 0$. The particle moves with constant acceleration $a$.




Using the graph or otherwise, derive an expression for $s$, the displacement of the particle at time $t$, in terms of $u$, $a$ and $t$.

Page for extra work.

Label any extra work clearly with the question number and part.



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Leaving Certificate – Ordinary Level

# Applied Mathematics

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Afternoon 2:00 - 4:30